AAS 25-879

A ROBUST METHOD FOR INITIAL ORBIT DETERMINATION USING RANGE-SWEPT SEEDING AND FULL-STATE DIFFERENTIAL CORRECTION

Daniel Parrott*

Initial orbit determination (IOD) remains a foundational problem in astrodynamics and space surveillance, forming the critical first step in orbit cataloging, track association, conjunction assessment, and long-term prediction. Classical IOD methods—such as Gauss's method, double-r iteration, and Lambert solvers—are well established and typically perform reliably when observational geometry and orbital motion are favorable. However, these techniques often fail when applied to highly eccentric Earth-orbiting objects, especially when observations are taken near apogee, where the satellite's apparent motion is slow and geometric leverage is limited. Such failure modes are frequently attributed to numerical instability in the solution of Kepler's equation and to the sensitivity of propagation techniques such as the f and g series, both of which are core dependencies in many traditional IOD frameworks. These limitations present a significant obstacle to reliable orbit determination in modern space situational awareness (SSA) environments, where high-eccentricity objects and sparse, geometry-challenged observations are increasingly common. Examples of such objects include geostationary transfer vehicles, Molniya-class satellites, decaying upper stages, and some debris fragments. These objects often exhibit low angular rates near apogee, poor observational leverage, and strong sensitivity to initial conditions—precisely the conditions under which many classical IOD methods become unreliable. Without a robust IOD capability, such objects are at risk of being lost after initial detection, preventing further tracking or follow-up observations.

INTRODUCTION

This paper introduces a robust and efficient method for initial orbit determination that is specifically well-suited for high-eccentricity artificial satellites, a scenario where classical methods are prone to failure. The method builds on a range-parameterized orbit seeding function (so2rv) that estimates an initial orbit from two optical observations by assigning a line-of-sight range variable "*rr*". For a given *rr* value, the method computes a candidate position and velocity vector that serves as a seed for a full six-parameter differential correction process. The differential corrector refines all classical orbital elements (semi-major axis, eccentricity, inclination, RAAN, argument of perigee, and mean anomaly) by minimizing angular residuals across all available observations. Each refined orbit is then scored using a root-mean-square error (RMSE) metric

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based on residuals in right ascension and declination. By sweeping through multiple plausible values of *rr*, and performing full-state differential correction at each step, the method selects the orbit with the lowest RMSE as the optimal solution.

This scoring-driven, range-swept approach ensures that the method does not depend on a single iteration of differential correction, thereby mitigating the risk of local minima or divergence. Importantly, the algorithm does not require solving Lambert's problem, nor does it rely on f and g series propagation, both of which can become numerically unstable in highly elliptical orbits. In contrast to the double r-iteration Lambert approach described by Pastor et al. (2021)[1], which solves for two independent range values at the endpoints of an observational arc and uses Lambert's solution to connect them, the proposed method treats range as a single control parameter. It refines not the geometry of the endpoints, but the entire orbital state vector through nonlinear least-squares optimization.

DEFINITION

Throughout this paper the term "Initial Orbit Determination" (or IOD) is used. One definition for IOD is provided by Vallado (2022)[2]: "We define the overall topic of initial orbit determination as the first-time development of orbital elements for a body in motion in space." Strictly speaking, therefore, one might consider only the orbits generated by a primitive routine such as "so2rv" to be in the category of IOD, while any refinement of such orbits could be considered regular "Orbit Determination" (OD). However, in this paper the term IOD is meant to refer to an orbit generated from observations made in a single night, regardless of the method used. In other words, we are computing an initial orbit from a short arc of observations (one night) of an object with the hope of recovering the same object on a subsequent night. This also mirrors the use of IOD employed in the Pastor paper, which "formulates the angles-only IOD problem as a boundary value problem in which pairs of ranges is estimated so that the residuals of the available observations are minimised." While the method described in this paper does not refine pairs of ranges, it does refine the range parameter used in "so2rv" in order to minimize the residuals of the available observations, therefore it would seem appropriate to similarly consider the method as initial orbit determination.

BACKGROUND

This work builds upon an orbit estimation framework originally developed by S. Campbell, specifically the "so2rv" and "so3rv" functions, which compute initial orbits from two and three optical observations, respectively. During experimental use of "so3rv", it was frequently observed to fail when the apparent angular motion between observations was slow – less than 600 arcseconds per minute – as typically occurs near apogee in highly eccentric orbits. However, "so2rv" consistently produced a candidate orbit even in these conditions, albeit with a tradeoff: since it relies on only two observations, it is more likely to converge to a suboptimal solution, particularly for non-circular orbits.

This observation led to the development of the present method: instead of relying on a single "so2rv" output, we introduced a tunable version that sweeps over a range of plausible line-of-sight distances (rr) to generate multiple seed orbits. Each candidate orbit is then refined using full-state differential correction, and the resulting refined orbits are evaluated by computing the root-mean-square error (RMSE) against the observed angular measurements. The orbit with the lowest RMSE is selected as the final result. We name this proposed method Range-Swept Differential Correction, or RSDC.

In contrast, the method by Pastor et al. performs differential correction on the front-end by adjusting the two unknown range values (r_1, r_2) used to generate the endpoints for Lambert's solution. While this approach is well-formulated and potentially effective, especially when the observation geometry is favorable, our method differs by avoiding Lambert's problem entirely. This structure has demonstrated reliable performance across a wide range of high-eccentricity cases, including those where classical approaches frequently fail.

The advantages of the method are threefold: (1) it maintains robustness under difficult geometric conditions, including cases where all observations occur near apogee, (2) it is straightforward to understand and apply the method (3) it is modular and adaptable to any observation set that includes at least two angular measurements. While, like any method, it requires a minimum amount of observational spread to constrain the orbit, it places no strict requirements on ideal observation spacing and has demonstrated reliable performance in scenarios where classical methods fail due to poor geometry or nonuniform observation timing.

ANALYSIS

Empirical validation has been conducted on a large dataset of high-eccentricity artificial satellites, with particular emphasis on cases where observations were taken near apogee. In every tested case, the proposed method successfully produced the desired orbit solution, while conventional IOD techniques typically produced suboptimal solutions due to ill-conditioned intermediate states. Five representative examples are included to demonstrate scenarios in which the observational geometry proves challenging for conventional IOD techniques. The observations for each of the five examples are also shown in the appendices.

In order to compare the new method against the traditional IOD techniques, each example computes the predicted position of the satellite 1 hour, 6 hours, 12 hours, and 24 hours from the time of the first observation, using one of five methods. This predicted position, which is computed from the orbit returned from the associated method, is then compared against the predicted position using the published orbit of the object. The difference is computed in terms of the along-track error, indicating how early or late the object was predicted to be within the expected coordinates.

Table 1 - Explanation of Methods

Method	Description
so2rv	Uses two angles-only observations to compute an orbit.
so3rv	Uses three angles-only observations to compute an orbit. Involves f and g series which can become numerically unstable under certain observation sets.
DC_so2rv	Uses single orbit computed from so2rv, refined with differential correction using additional observations from same night.
DC_so3rv	Uses single orbit computed from so3rv, refined with differential correction using additional observations from same night.
RSDC	Range-swept differential correction. Generates multiple orbits from so2rv, using tunable range parameter. Each orbit is refined using additional observations from same night. Orbit with the lowest RMSE is selected as the output. Technical note: differential correction itself iterates through multiple candidate orbits; the difference here is that those orbits are a result of exploring a multi-dimensional Newton-Raphson iteration, while the multiple orbits explored via

	RSDC is through the result of exploring different line-of-sight distances (ranges). In effect, differential correction is being applied to each candidate orbit, rather than to just a single orbit as is the case with the other methods.

Table 2 - MERIDIAN 4 (e=0.736), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	<1 min	>60 min	>60 min	>60 min
so3rv	8 min	>60 min	>60 min	>60 min
DC_so2rv	<1 min	53 min	>60 min	>60 min
DC_so3rv	<1 min	>60 min	>60 min	>60 min
RSDC	<1 min	9 min	14 min	46 min

Table 3 - OPS 7724 (e=0.726), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	10 min	>60 min	>60 min	>60 min
so3rv	<1 min	>60 min	>60 min	>60 min
DC_so2rv	10 min	>60 min	>60 min	>60 min
DC_so3rv	<1 min	>60 min	>60 min	>60 min
RSDC	<1 min	4 min	5 min	11 min

Table 4 - MOLNIYA 3-50 (e=0.681), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	2 min	>60 min	>60 min	>60 min
so3rv	n/a	n/a	n/a	n/a
DC_so2rv	2 min	>60 min	>60 min	>60 min
DC_so3rv	n/a	n/a	n/a	n/a
RSDC	<1 min	2 min	6 min	16 min

Table 5 - MERIDIAN 10 (e=0.674), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	1 min	14	>60 min	>60 min
so3rv	n/a	n/a	n/a	n/a
DC_so2rv	<1 min	>60 min	>60 min	>60 min

DC_so3rv	n/a	n/a	n/a	n/a
RSDC	<1 min	1 min	1 min	4 min

Table 6 - POLAR (e=0.613), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	<1 min	>60 min	>60 min	>60 min
so3rv	n/a	n/a	n/a	n/a
DC_so2rv	<1 min	19 min	>60 min	>60 min
DC_so3rv	n/a	n/a	n/a	n/a
RSDC	<1 min	3	24	35

SYNTHETIC OBSERVATIONS

The five examples shown in tables 2-6 were generated using measurements from commercial off-the-shelf equipment: Celestron RASA 8, ZWO 2600MM camera, and ZWO AM5 mount. The camera was operated in bin2 mode, yielding a plate scale of 3.88"/pixel (slightly under-sampled). Furthermore, while a GPS receiver was used to discipline the computer clock at the beginning of the imaging session, it was noted that the USB connection to the mount would drop if the GPS connection (also USB) remained active – so throughout the imaging session no additional corrections were made to the computer clock. All this is to say, if we wish to evaluate the performance of the various methods under ideal conditions, we should instead make use of "synthetic" observations that are generated using the predicted position of the object according to its published orbit. Doing so allows us to see how well these methods can perform when supplied with the best possible measurements. Additionally, the observation arc used throughout this section will consistently be the same: 45 minutes, which is appropriate for the slow movement of the objects at apogee.

Table 7 - MERIDIAN 4 (e=0.736), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	<1 min	>60 min	>60 min	>60 min
so3rv	9 min	>60 min	>60 min	>60 min
DC_so2rv	<1 min	>60 min	>60 min	>60 min
DC_so3rv	<1 min	>60 min	>60 min	>60 min
RSDC	<1 min	1 min	1 min	4 min

Table 8 - OPS 7724 (e=0.726), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	4 min	>60 min	>60 min	>60 min

so3rv	<1 min	7 min	11 min	28 min
DC_so2rv	4 min	>60 min	>60 min	>60 min
DC_so3rv	<1 min	7 min	12 min	29 min
RSDC	<1 min	<1 min	1 min	2 min

Table 9 - MOLNIYA 3-50 (e=0.681), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	3 min	>60 min	>60 min	>60 min
so3rv	3 min	>60 min	>60 min	>60 min
DC_so2rv	4 min	>60 min	>60 min	>60 min
DC_so3rv	<1 min	12 min	33 min	>60 min
RSDC	<1 min	<1 min	1 min	2 min

Table 10 - MERIDIAN 10 (e=0.674), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	3 min	>60 min	>60 min	>60 min
so3rv	n/a	n/a	n/a	n/a
DC_so2rv	3 min	>60 min	>60 min	>60 min
DC_so3rv	n/a	n/a	n/a	n/a
RSDC	<1 min	1 min	2 min	6 min

Table 11 - POLAR (e=0.613), Along-track error (in minutes) after Elapsed Time

Method	1 hr	6 hrs	12 hrs	24 hrs
so2rv	1 min	>60 min	>60 min	>60 min
so3rv	n/a	n/a	n/a	n/a
DC_so2rv	<1 min	12 min	36 min	38 min
DC_so3rv	n/a	n/a	n/a	n/a
RSDC	<1 min	<1 min	2 min	4 min

Tables 7-11 show the results of using synthetic observations. As before, RSDC outperforms the other IOD methods. Here, RSDC exhibits a maximum along-track error of 6 minutes after 24 hours. By comparison, the other methods exhibit, at best, an along-track error of 28 minutes.

Figure 1 shows a visual comparison of POLAR (SATCAT number 23802), at T=0 and T=24 hours from first observation. Three orbits are plotted in both scenarios: the published orbit, the computed orbit from DC_so2rv (which had the best fit from the "traditional" IOD methods for this object), and the computed orbit from RSDC.

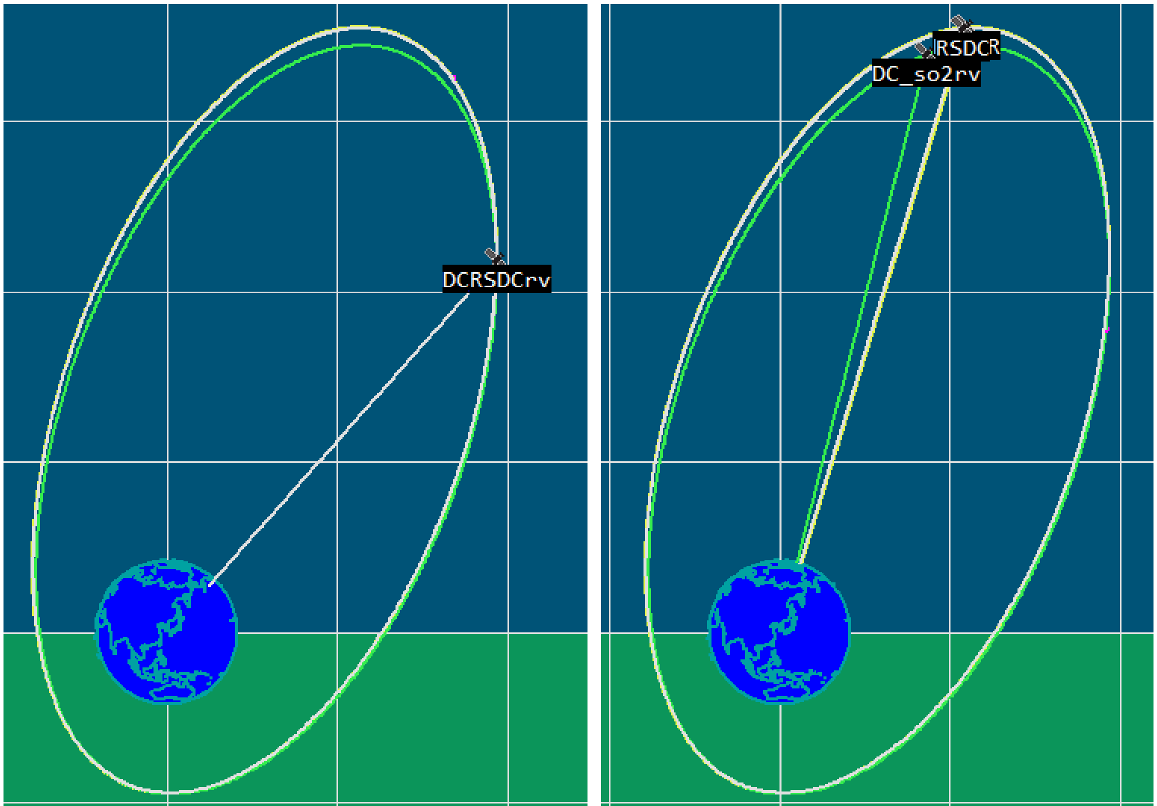

Figure 1 - Orbit Comparison. Left: T=0 hours Right: T=24 hours.

As the labels are prone to overlap when zoomed out, Figure 2 presents a zoomed-in view of the same three orbits, at T=24 hours. Here it can be seen that the orbit computed using the RSDC method is in very close agreement with the published orbit, relative to the orbit from DC_so2rv.

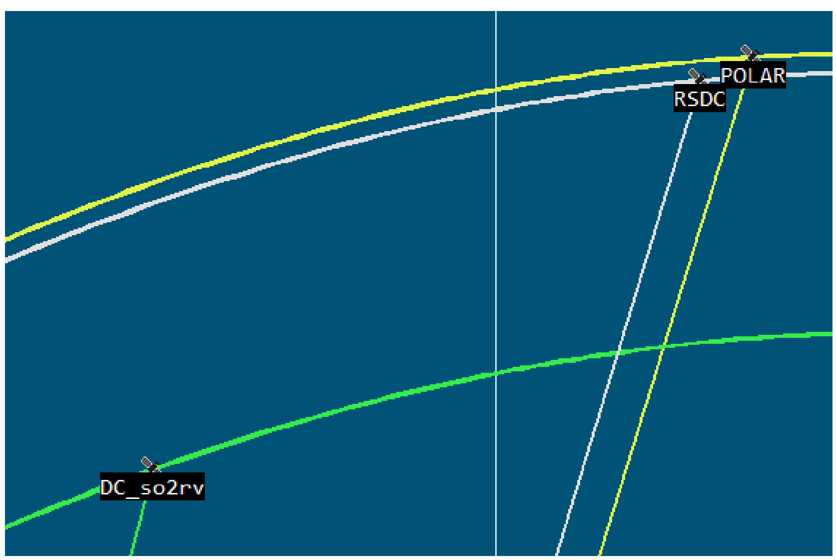

Figure 2 - Orbit Comparison, T=24 hours, zoomed in.

RMSE SCORE BEHAVIOR

In this section we examine the behavior of the RMSE scoring function to assess the stability of the RSDC approach. We do this for both the real observations as well as the synthetic observations. *MERIDIAN 4* is used for the example. Figure 3 shows both a zoomed-out as well as a zoomed-in view of the RMSE score as a function of the range (*rr*) parameter, using the set of real observations. The range units are in Earth radii (~6378 km). Approximately 800 candidate orbits were generated, one for each range value, in intervals of 0.01 Earth radii (or 63.78 km). The whole process of generating the orbits, refining each orbit via differential correction, and scoring the refined orbits takes less than 10 seconds for all 800 candidate orbits.

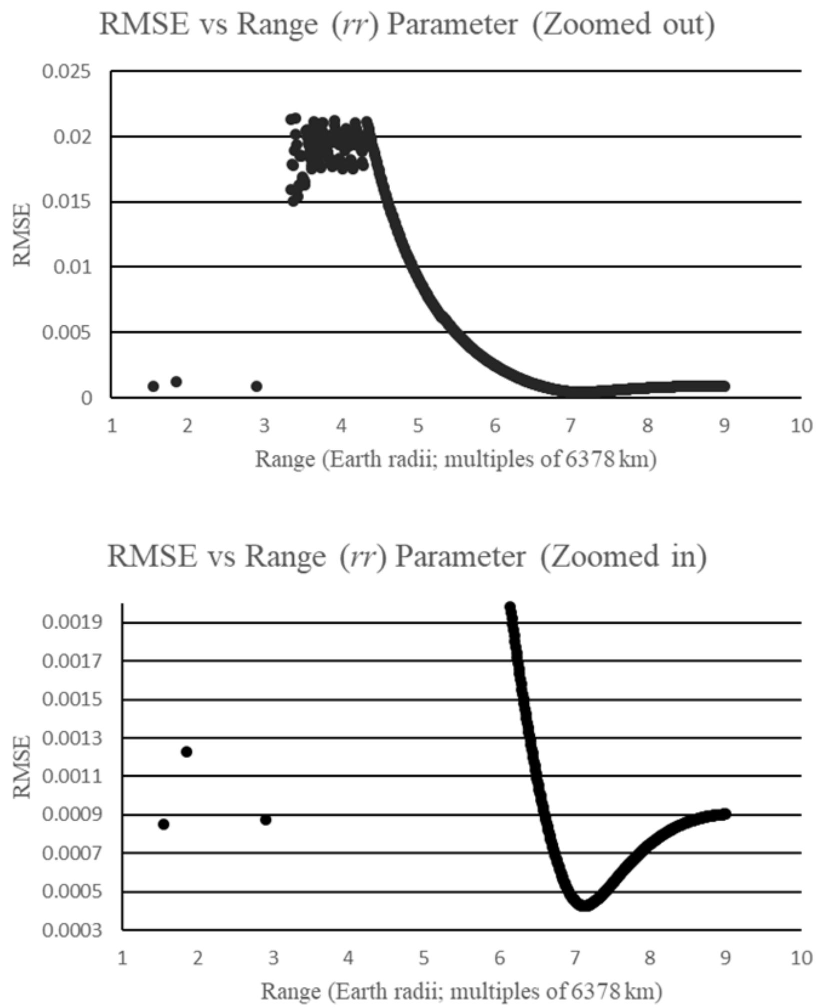

Figure 3 - RMSE vs Range (Real Observations)

Figure 4 shows a similar set of graphs, but this time using the set of synthetic observations. In both cases, we can see that the RSDC approach gravitates towards a clearly defined global minimum across the entire span of the 800 candidate orbits, where each orbit was generated using the *so2rv* function with a different line-of-sight range (*rr*) value. To see how this behavior produces the desired orbital elements, we can inspect two elements: eccentricity (*e*) and semi-major axis (*a*). The latter element is inspected by proxy via the "revolutions per day" quantity of the two-line element set. Figure 5 shows two graphs, one depicting RMSE vs eccentricity and the other depicting RMSE vs Revs/Day, for the set of real observations. Finally, Figure 6 shows the same two graphs, but this time for the set of synthetic observations.

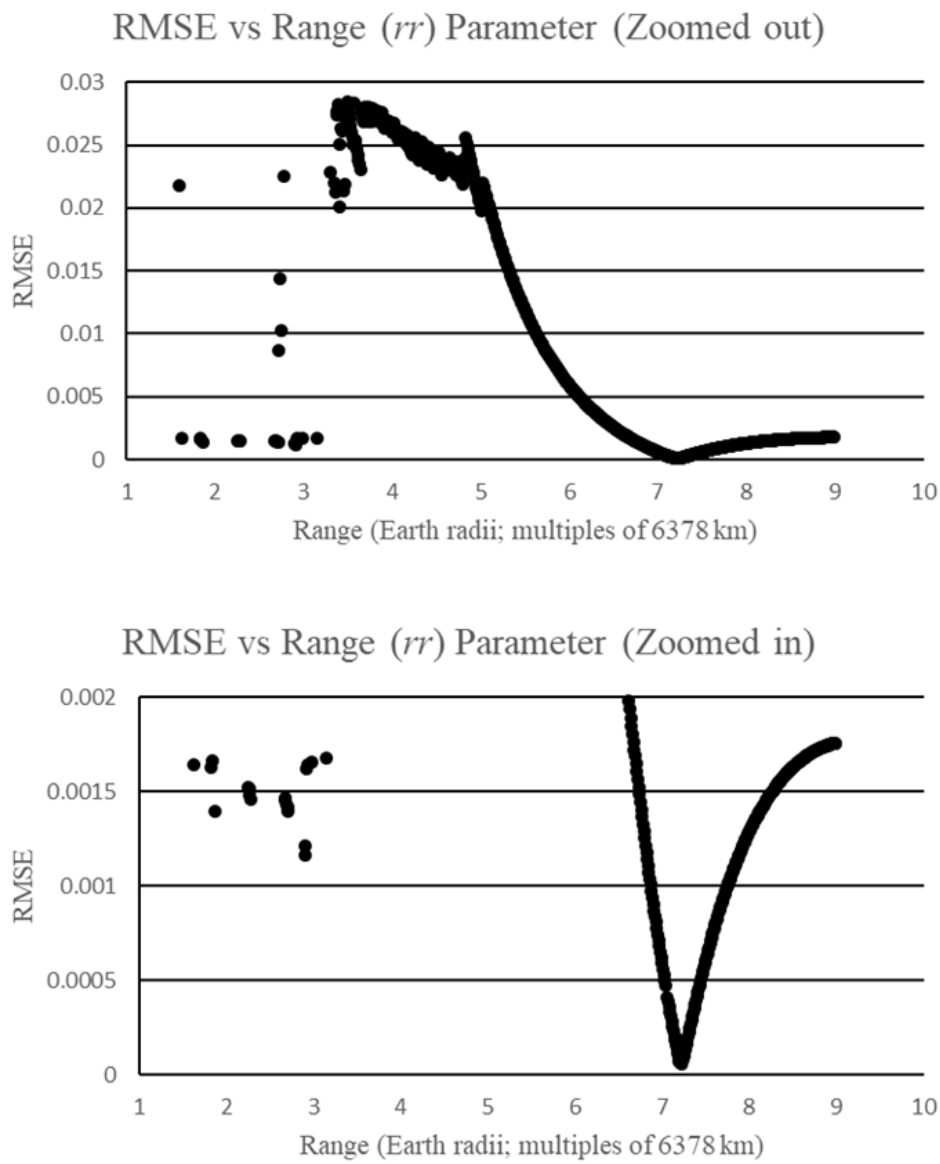

Figure 4 - RMSE vs Range (Synthetic Observations)

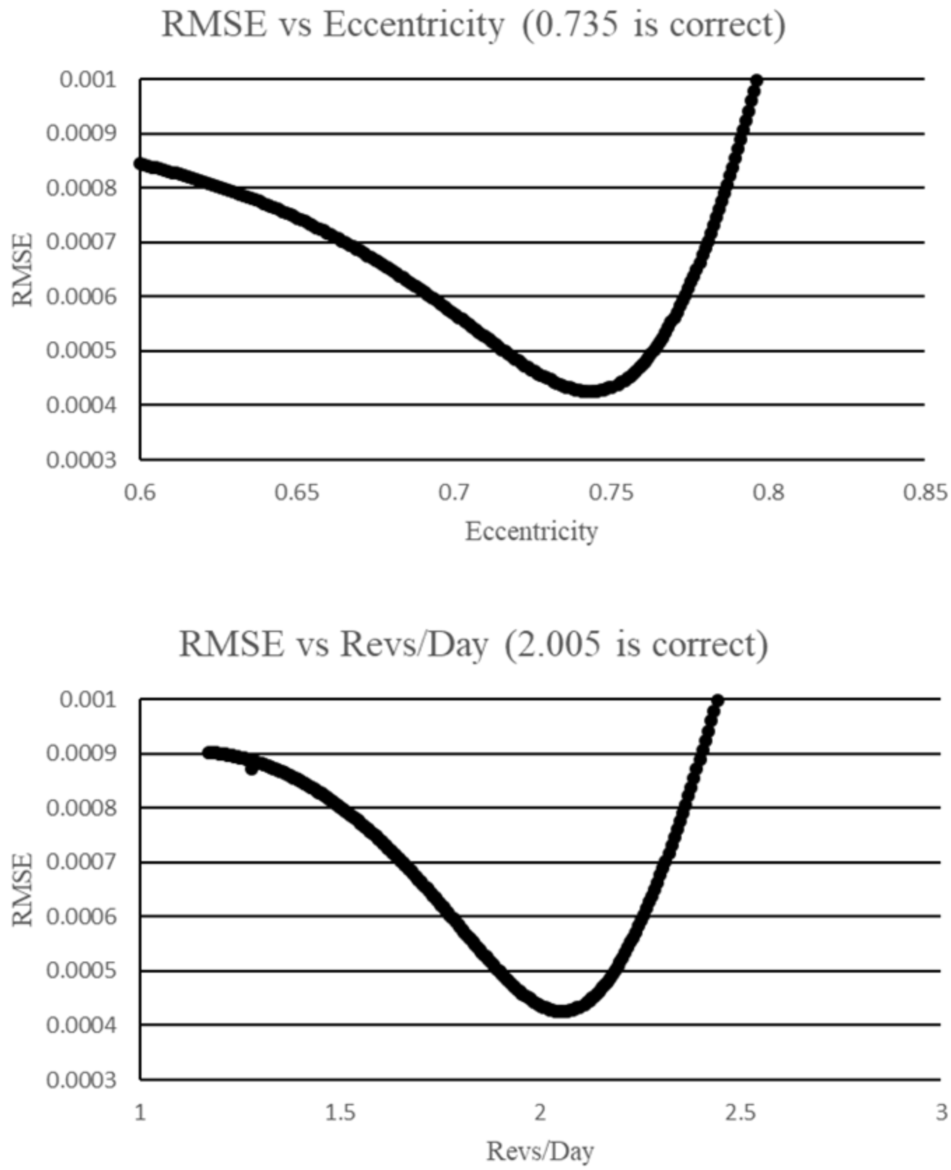

Figure 5 – Top: RMSE vs Eccentricity; Bottom: RMSE vs Revs/Day (Real Observations)

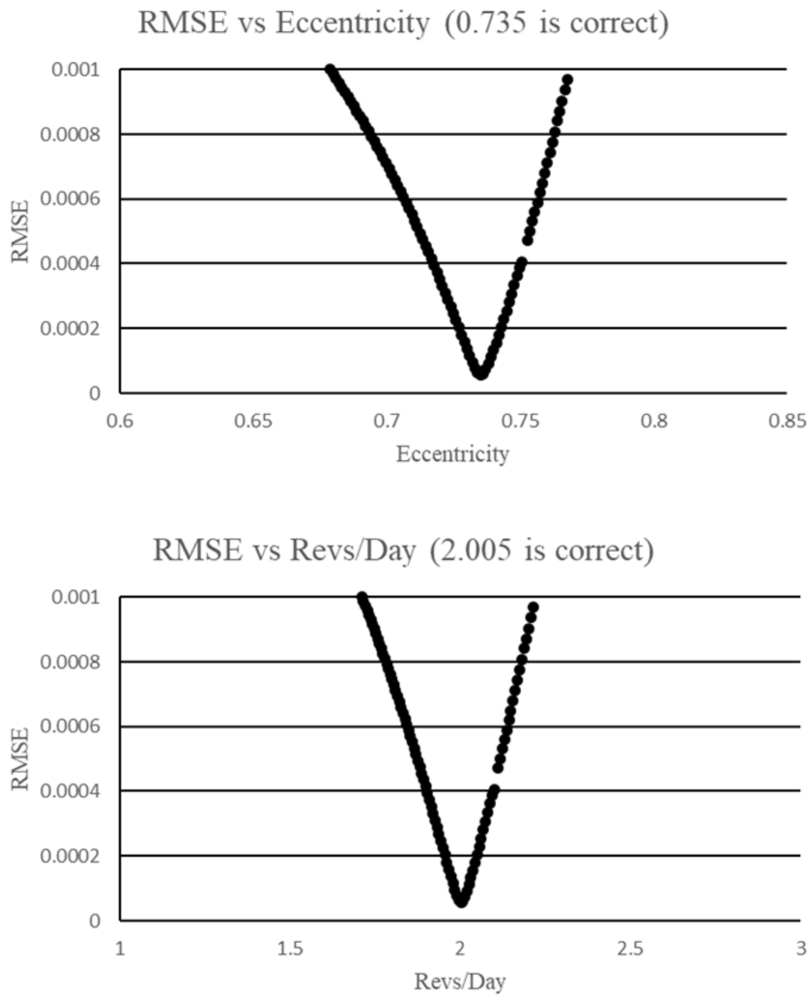

Figure 6 - Top: RMSE vs Eccentricity; Bottom: RMSE vs Revs/Day (Synthetic Observations)

CONCLUSION

This work contributes to the field of astrodynamics by offering a highly reliable and easily implementable alternative to traditional IOD methods. It is particularly relevant to operators of space surveillance networks, catalog maintenance systems, and any real-time tracking applications where observation gaps, high-eccentricity motion, or near-apogee geometries are present. A comparative performance analysis with respect to traditional IOD techniques is presented, along with a representative set of high-eccentricity test cases. Supporting tables and figures illustrate the behavior of the RMSE score as a function of the rr parameter, the convergence pattern of the differential corrector, and the relative accuracy of resulting orbital elements.

REFERENCES

[1]A. Pastor, M. Sanjurjo-Rivo, and D. Escobar, "Initial Orbit Determination Methods for Track-to-Track Association." *Advances in Space Research*. Vol. 68, No. 7, October 2021, pp. 2677–2694.

[2]D. Vallado, "Fundamentals of Astrodynamics and Applications." *Space Library Technology*. Microcosm Press, Fifth Edition. 2022.

APPENDIX A: MEASUREMENTS AND ORBITS OF *MERIDIAN 4*

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(REAL) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
37398          9001 G 20250610041202492 56 25 1325098+643785 17 S+149 10
37398          9001 G 20250610041212994 56 25 1325243+643820 17 S+141 10
37398          9001 G 20250610041221993 56 25 1325377+643852 17 S+147 10
37398          9001 G 20250610041230995 56 25 1325502+643877 17 S+148 10

37398          9001 G 20250610041912526 56 25 1331238+645128 17 S+114 10
37398          9001 G 20250610041922526 56 25 1331380+645158 17 S+149 10
37398          9001 G 20250610041940031 56 25 1331639+645210 17 S+145 10
37398          9001 G 20250610042000027 56 25 1331927+645271 17 S+155 10

37398          9001 G 20250610042257033 56 25 1334491+645785 17 S+140 10
37398          9001 G 20250610042317540 56 25 1334787+645844 17 S+143 10
37398          9001 G 20250610042322042 56 25 1334855+645855 17 S+148 10
37398          9001 G 20250610042338542 56 25 1335100+645906 17 S+149 10
37398          9001 G 20250610042356543 56 25 1335360+645956 17 S+152 10

37398          9001 G 20250610044313135 56 25 1352673+652799 17 S+149 10
37398          9001 G 20250610044331146 56 25 1352952+652835 17 S+229 10
37398          9001 G 20250610044353647 56 25 1353294+652883 17 S+232 10
37398          9001 G 20250610044409148 56 25 1353535+652915 17 S+143 10
37398          9001 G 20250610044412649 56 25 1353589+652920 17 S+139 10

37398          9001 G 20250610044551653 56 25 1355114+653120 17 S+146 10
37398          9001 G 20250610044605659 56 25 1355327+653149 17 S+138 10
37398          9001 G 20250610044615160 56 25 1355476+653167 17 S+144 10
37398          9001 G 20250610044626168 56 25 1355652+653194 17 S+126 10
37398          9001 G 20250610044628658 56 25 1355687+653191 17 S+145 10
37398          9001 G 20250610044650165 56 25 1356031+653238 17 S+140 10
------ end ------

ORBITS (FROM REAL MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
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1 37398U 11018A   25159.44908954 -.00000273  00000-0 -26845-2 0  9997
2 37398  62.1159 134.2596 7355930 260.8706  17.3469  2.00549376103276

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 37398U 0000000  25161.19919172 0.00000087  00000-0  50000-4 0    01
2 37398  62.5145 134.3907 6556886 265.2837 179.9981  1.62497674    09

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 37398U 0000000  25161.19919172 0.00000087  00000-0  50000-4 0    01
2 37398  62.4889 134.0008 6566455 260.2328 198.8555  1.61961051    06

ORBIT COMPUTED VIA so3rv:
0 USER_TLE
1 37398U 0000000  25161.18284190 0.00000087  00000-0  50000-4 0    05
2 37398  63.8530 133.4762 3866552 105.3163 350.5855  0.18096150    08
```

```
ORBIT COMPUTED VIA DC_so3rv:
0 REFINED_TLE
1 37398U 0000000  25161.18284190 0.00000087  00000-0  50000-4 0    05
2 37398  64.0298 133.6764 3760185 101.3188 351.6558  0.18355676    07

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 37398U 0000000  25161.19667980 0.00000087  00000-0  50000-4 0    08
2 37398  62.0650 134.0070 7436365 260.9789 199.1614  2.05463640    02

(SYNTHETIC) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
37398          9001 G 20250610041205004 56 25 1325162+643833 56 S+140 10
37398          9001 G 20250610041305004 56 25 1326011+644026 56 S+140 10
37398          9001 G 20250610041405004 56 25 1326864+644218 56 S+140 10
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37398          9001 G 20250610042705004 56 25 1338146+650511 56 S+140 10
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37398          9001 G 20250610044205005 56 25 1351652+652686 56 S+140 10
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ORBITS (FROM SYNTHETIC MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
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2 37398  62.1159 134.2596 7355930 260.8706  17.3469  2.00549376103276

ORBIT COMPUTED VIA so2rv:
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1 37398U 0000000  25161.20700236 0.00000087  00000-0  50000-4 0    02
2 37398  62.5294 134.4850 6547416 266.5160 179.9986  1.62506973    02

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 37398U 0000000  25161.20700236 0.00000087  00000-0  50000-4 0    02
2 37398  62.4890 133.9351 6564217 259.7772 205.1376  1.61567979    01

ORBIT COMPUTED VIA so3rv:
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1 37398U 0000000  25161.18616903 0.00000087  00000-0  50000-4 0    06
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ORBIT COMPUTED VIA DC_so3rv:
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1 37398U 0000000  25161.18616903 0.00000087  00000-0  50000-4 0    06
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ORBIT COMPUTED VIA RSDC:
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2 38826  62.1192 133.9933 7350336 260.8812 198.5178  2.00151176    07
```

APPENDIX B: MEASUREMENTS AND ORBITS OF *OPS 7724*

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(REAL) MEASUREMENTS:

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------ IOD Lines ------
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6791           9001 G 20250630030401950 56 25 1425246+530684 17 S+124 10
6791           9001 G 20250630030540809 56 25 1426395+525382 17 S+122 10
6791           9001 G 20250630030606188 56 25 1426691+525048 17 S+130 10
6791           9001 G 20250630030631847 56 25 1426988+524707 17 S+147 10
6791           9001 G 20250630030914619 56 25 1428877+522514 17 S+134 10
6791           9001 G 20250630030939130 56 25 1429161+522182 17 S+111 10
6791           9001 G 20250630031005132 56 25 1429465+521821 17 S+119 10
6791           9001 G 20250630031730567 56 25 1434640+511501 17 S+120 10
6791           9001 G 20250630031845080 56 25 1435507+510403 17 S+127 10
6791           9001 G 20250630032000086 56 25 1436378+505285 17 S+135 10
6791           9001 G 20250630032330605 56 25 1438830+502078 17 S+123 10
6791           9001 G 20250630032445114 56 25 1439698+500915 17 S+116 10
6791           9001 G 20250630032600120 56 25 1440574+495732 17 S+132 10
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ORBITS (FROM REAL MEASUREMENTS):

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1  6791U 73056A   25177.78625028  .00000375  00000-0 -24061-2 0  9995
2  6791  62.2539  78.2006 7255941 277.2451  12.7050  2.00639858 42479

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 06791U 0000000  25181.14305694 0.00000087  00000-0  50000-4 0    09
2 06791  63.5825  75.8208 1687879 303.7278 180.0324  0.95374411    02

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 06791U 0000000  25181.14305694 0.00000087  00000-0  50000-4 0    09
2 06791  63.5828  75.8202 1690804 303.6647 180.1208  0.95359857    09

ORBIT COMPUTED VIA so3rv:
0 USER_TLE
1 06791U 0000000  25181.13141920 0.00000087  00000-0  50000-4 0    08
2 06791  61.8227  78.4422 7661440 283.2026 256.8783  2.52036129    08

ORBIT COMPUTED VIA DC_so3rv:
0 REFINED_TLE
1 06791U 0000000  25181.13141920 0.00000087  00000-0  50000-4 0    08
2 06791  61.8085  78.4277 7657965 283.2074 256.8119  2.52523553    03

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 06791U 0000000  25181.13802176 0.00000087  00000-0  50000-4 0    05
2 06791  62.2634  77.7088 7228698 277.1332 273.5957  1.99264516    03

(SYNTHETIC) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
```

```
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
6791            9001 G 20250630030305005 56 25 1424585+531408 56 S+140 10
6791            9001 G 20250630030405004 56 25 1425281+530628 56 S+140 10
6791            9001 G 20250630030505004 56 25 1425977+525842 56 S+140 10
6791            9001 G 20250630031705005 56 25 1434331+511860 56 S+140 10
6791            9001 G 20250630031805005 56 25 1435029+510979 56 S+140 10
6791            9001 G 20250630031905005 56 25 1435726+510090 56 S+140 10
6791            9001 G 20250630033205005 56 25 1444833+485746 56 S+140 10
6791            9001 G 20250630033305005 56 25 1445538+484730 56 S+140 10
6791            9001 G 20250630033405005 56 25 1446243+483705 56 S+140 10
6791            9001 G 20250630034705005 56 25 1455497+461355 56 S+140 10
6791            9001 G 20250630034805005 56 25 1456217+460166 56 S+140 10
6791            9001 G 20250630034905004 56 25 1456938+454963 56 S+140 10
------ end ------

ORBITS (FROM SYNTHETIC MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 OPS 7724
1  6791U 73056A   25177.78625028  .00000375  00000-0 -24061-2 0  9995
2  6791  62.2539  78.2006 7255941 277.2451  12.7050  2.00639858 42479

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 06791U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    00
2 06791  63.9196  75.1657 0227928 308.4473 180.2297  0.77909187    06

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 06791U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    00
2 06791  63.9196  75.1657 0227928 308.4473 180.2297  0.77909187    06

ORBIT COMPUTED VIA so3rv:
0 USER_TLE
1 06791U 0000000  25181.13825237 0.00000087  00000-0  50000-4 0    08
2 06791  62.2957  77.7094 7230745 276.7299 274.9168  1.97168909    07

ORBIT COMPUTED VIA DC_so3rv:
0 REFINED_TLE
1 06791U 0000000  25181.13825237 0.00000087  00000-0  50000-4 0    08
2 06791  62.2871  77.6613 7231957 276.7703 274.8663  1.97060293    00

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 06791U 0000000  25181.13755793 0.00000087  00000-0  50000-4 0    07
2 06791  62.2563  77.7109 7258574 277.3140 273.3577  2.00831073    00
```

APPENDIX C: MEASUREMENTS AND ORBITS OF *MOLNIYA 3-50*

```
(REAL) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
25847           9001 G 20250630041740601 56 25 1655727+644683 17 S+142 10
25847           9001 G 20250630041855113 56 25 1657524+645286 17 S+148 10
25847           9001 G 20250630042010123 56 25 1659337+645881 17 S+148 10
25847           9001 G 20250630043001180 56 25 1713621+654143 17 S+138 10
```

```
25847           9001 G 20250630043115688 56 25 1715415+654628 17 S+137 10
25847           9001 G 20250630043230696 56 25 1717234+655104 17 S+153 10
25847           9001 G 20250630044219257 56 25 1731441+662472 17 S+143 10
25847           9001 G 20250630044333772 56 25 1733234+662851 17 S+146 10
25847           9001 G 20250630044448778 56 25 1735038+663222 17 S+130 10
25847           9001 G 20250630045618347 56 25 1751574+670182 17 S+139 10
25847           9001 G 20250630045730355 56 25 1753293+670445 17 S+139 10
25847           9001 G 20250630045845367 56 25 1755081+670708 17 S+145 10
------ end ------

ORBITS (FROM REAL MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 MOLNIYA 3-50
1 25847U 99036A   25178.07657039 -.00000025  00000-0 -97851-1 0  9993
2 25847  63.5175 192.1379 6812602 279.9780  15.0092  2.00627567190297

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 25847U 0000000  25181.20746953 0.00000087  00000-0  50000-4 0    06
2 25847  64.1569 190.0918 2567907 263.6528 179.9952  1.06295592    06

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 25847U 0000000  25181.20746953 0.00000087  00000-0  50000-4 0    06
2 25847  64.1960 190.3206 2553681 263.1056 180.7339  1.06419990    08

ORBIT COMPUTED VIA so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA DC_so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 25847U 0000000  25181.18924417 0.00000087  00000-0  50000-4 0    06
2 25847  63.5286 191.7874 6785230 280.2030 102.9427  1.98819187    08

(SYNTHETIC) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
25847           9001 G 20250630030305005 56 25 1509027+531399 56 S+140 10
25847           9001 G 20250630030405004 56 25 1510461+532967 56 S+140 10
25847           9001 G 20250630030505004 56 25 1511893+534508 56 S+140 10
25847           9001 G 20250630031705005 56 25 1529004+563140 56 S+140 10
25847           9001 G 20250630031805005 56 25 1530425+564386 56 S+140 10
25847           9001 G 20250630031905005 56 25 1531847+565612 56 S+140 10
25847           9001 G 20250630033205005 56 25 1550326+591934 56 S+140 10
25847           9001 G 20250630033305005 56 25 1551749+592922 56 S+140 10
25847           9001 G 20250630033405005 56 25 1553172+593895 56 S+140 10
25847           9001 G 20250630034705005 56 25 1611725+613299 56 S+140 10
25847           9001 G 20250630034805005 56 25 1613157+614088 56 S+140 10
25847           9001 G 20250630034905004 56 25 1614589+614865 56 S+140 10
------ end ------

ORBITS (FROM SYNTHETIC MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
```

```
0 MOLNIYA 3-50
1 25847U 99036A   25178.07657039 -.00000025  00000-0 -97851-1 0  9993
2 25847  63.5175 192.1379 6812602 279.9780  15.0092  2.00627567190297

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 38826U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    04
2 38826  63.7339 190.5314 0001361 276.2156 154.6420  1.00190993    07

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 38826U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    04
2 38826  64.6298 192.5279 0063635 257.4847 172.6222  1.00184910    05

ORBIT COMPUTED VIA so3rv:
0 USER_TLE
1 38826U 0000000  25181.13825237 0.00000087  00000-0  50000-4 0    02
2 38826  63.5943 192.0640 7222014 281.4309  56.0945  1.81244667    04

ORBIT COMPUTED VIA DC_so3rv:
0 REFINED_TLE
1 38826U 0000000  25181.13825237 0.00000087  00000-0  50000-4 0    02
2 38826  63.4619 191.7857 6854814 278.2115  68.8996  2.11397422    05

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 38826U 0000000  25181.12853014 0.00000087  00000-0  50000-4 0    05
2 38826  63.5190 191.8186 6810254 280.0063  59.2926  2.00331171    03
```

APPENDIX D: MEASUREMENTS AND ORBITS OF *MERIDIAN 10*

```
(REAL) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
52145           9001 G 20250630042205629 56 25 1556505+641389 17 S+144 10
52145           9001 G 20250630042320138 56 25 1557981+641869 17 S+127 10
52145           9001 G 20250630042435150 56 25 1559459+642346 17 S+134 10
52145           9001 G 20250630043403706 56 25 1610752+645682 17 S+142 10
52145           9001 G 20250630043518214 56 25 1612243+650086 17 S+134 10
52145           9001 G 20250630043633223 56 25 1613739+650485 17 S+140 10
52145           9001 G 20250630044634782 56 25 1625827+653421 17 S+135 10
52145           9001 G 20250630044749299 56 25 1627328+653752 17 S+127 10
52145           9001 G 20250630044904304 56 25 1628849+654079 17 S+147 10
52145           9001 G 20250630050000371 56 25 1642159+660629 17 S+132 10
52145           9001 G 20250630050109883 56 25 1643575+660868 17 S+145 10
52145           9001 G 20250630050224892 56 25 1645103+661120 17 S+134 10
------ end ------

ORBITS (FROM REAL MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 MERIDIAN 10
1 52145U 22030A   25180.04239168  .00000223  00000-0  00000-0 0  9990
2 52145  62.6609 177.9835 6743975 273.3745  17.7634  2.00601496 23944

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
```

```
1 52145U 0000000  25181.21001032 0.00000087  00000-0  50000-4 0    00
2 52145  63.1241 176.1008 4191218 263.7344 179.9967  1.28630002    04

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 52145U 0000000  25181.21001032 0.00000087  00000-0  50000-4 0    00
2 52145  63.4458 177.9544 5041293 294.0402 105.4718  1.10704574    06

ORBIT COMPUTED VIA so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA DC_so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 52145U 0000000  25181.19205119 0.00000087  00000-0  50000-4 0    09
2 52145  62.6541 177.8417 6749820 273.3054 128.2072  2.01087423    07

(SYNTHETIC) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
52145          9001 G 20250630030305005 56 25 1427029+554255 56 S+140 10
52145          9001 G 20250630030405004 56 25 1428126+555234 56 S+140 10
52145          9001 G 20250630030505004 56 25 1429224+560202 56 S+140 10
52145          9001 G 20250630031705005 56 25 1442426+574978 56 S+140 10
52145          9001 G 20250630031805005 56 25 1443530+575811 56 S+140 10
52145          9001 G 20250630031905005 56 25 1444635+580634 56 S+140 10
52145          9001 G 20250630033205005 56 25 1459076+594532 56 S+140 10
52145          9001 G 20250630033305005 56 25 1500193+595235 56 S+140 10
52145          9001 G 20250630033405005 56 25 1501312+595930 56 S+140 10
52145          9001 G 20250630034705005 56 25 1515965+612290 56 S+140 10
52145          9001 G 20250630034805005 56 25 1517101+612883 56 S+140 10
52145          9001 G 20250630034905004 56 25 1518238+613470 56 S+140 10
------ end ------

ORBITS (FROM SYNTHETIC MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 MERIDIAN 10
1 52145U 22030A   25180.04239168  .00000223  00000-0  00000-0 0  9990
2 52145  62.6609 177.9835 6743975 273.3745  17.7634  2.00601496 23944

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 52145U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    04
2 52145  63.1089 176.0568 0001157 274.7119 157.8820  0.76775849    08

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 52145U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    04
2 52145  63.1089 176.0568 0001157 274.7119 157.8820  0.76775849    08

ORBIT COMPUTED VIA so3rv:
so3rv failed to compute an orbit.
ORBIT COMPUTED VIA DC_so3rv:
so3rv failed to compute an orbit.
```

```
ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 52145U 0000000  25181.13755793 0.00000087  00000-0  50000-4 0    01
2 52145  62.6655 177.8706 6737892 273.4825  88.5052  1.99931407    09
```

APPENDIX E: MEASUREMENTS AND ORBITS OF *POLAR*

```
(REAL) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
23802          9001 G 20250630034718877 56 25 1538178+194839 17 S+150 10
23802          9001 G 20250630034801365 56 25 1538173+195394 17 S+122 10
23802          9001 G 20250630034844134 56 25 1538168+195947 17 S+120 10
23802          9001 G 20250630035047928 56 25 1538152+201550 17 S+140 10
23802          9001 G 20250630035202441 56 25 1538142+202512 17 S+131 10
23802          9001 G 20250630035317443 56 25 1538135+203478 17 S+139 10
23802          9001 G 20250630040059993 56 25 1538079+213360 17 S+133 10
23802          9001 G 20250630040214503 56 25 1538068+214299 17 S+131 10
23802          9001 G 20250630040329519 56 25 1538062+215241 17 S+137 10
23802          9001 G 20250630041202570 56 25 1538001+225598 17 S+135 10
23802          9001 G 20250630041317072 56 25 1537993+230514 17 S+123 10
23802          9001 G 20250630041432080 56 25 1537988+231433 17 S+146 10
23802          9001 G 20250630043810229 56 25 1537844+260286 17 S+125 10
23802          9001 G 20250630043924746 56 25 1537839+261148 17 S+136 10
23802          9001 G 20250630044039749 56 25 1537831+262010 17 S+131 10
23802          9001 G 20250630045221822 56 25 1537778+273999 17 S+131 10
23802          9001 G 20250630045336329 56 25 1537773+274838 17 S+123 10
23802          9001 G 20250630045451338 56 25 1537768+275677 17 S+137 10
------ end ------

ORBITS (FROM REAL MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 POLAR
1 23802U 96013A   25180.28201142  .00000165  00000-0  00000-0 0  9992
2 23802  79.4568 230.9222 6134304 220.9874  71.3808  1.29846837140278

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 23802U 0000000  25181.20476086 0.00000087  00000-0  50000-4 0    02
2 23802  78.4023 230.4654 6563223 209.6218 179.9830  1.60840450    05

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 23802U 0000000  25181.20476086 0.00000087  00000-0  50000-4 0    02
2 23802  78.7232 230.6002 6705861 216.8674 152.7990  1.52885818    05

ORBIT COMPUTED VIA so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA DC_so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 23802U 0000000  25181.16822341 0.00000087  00000-0  50000-4 0    06
2 23802  79.5702 230.9587 6031009 221.7555 124.4903  1.26159936    02
```

```
(SYNTHETIC) MEASUREMENTS:

------ Station ------
# No ID  Latitude    Longitude  Elev  Observer
9001 FL   35.410000  -97.670000  390  D. Parrott
------ IOD Lines ------
23802          9001 G 20250630030305005 56 25 1538511+134066 56 S+140 10
23802          9001 G 20250630030405004 56 25 1538504+134949 56 S+140 10
23802          9001 G 20250630030505004 56 25 1538497+135829 56 S+140 10
23802          9001 G 20250630031705005 56 25 1538408+154192 56 S+140 10
23802          9001 G 20250630031805005 56 25 1538401+155040 56 S+140 10
23802          9001 G 20250630031905005 56 25 1538393+155885 56 S+140 10
23802          9001 G 20250630033205005 56 25 1538295+174660 56 S+140 10
23802          9001 G 20250630033305005 56 25 1538287+175474 56 S+140 10
23802          9001 G 20250630033405005 56 25 1538280+180284 56 S+140 10
23802          9001 G 20250630034705005 56 25 1538182+194635 56 S+140 10
23802          9001 G 20250630034805005 56 25 1538174+195416 56 S+140 10
23802          9001 G 20250630034905004 56 25 1538167+200196 56 S+140 10
------ end ------

ORBITS (FROM SYNTHETIC MEASUREMENTS):

TRUTH DATA (PUBLISHED ORBIT):
0 POLAR
1 23802U 96013A   25180.28201142  .00000165  00000-0  00000-0 0  9992
2 23802  79.4568 230.9222 6134304 220.9874  71.3808  1.29846837140278

ORBIT COMPUTED VIA so2rv:
0 USER_TLE
1 23802U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    02
2 23802  79.6556 230.9876 5693099 202.2535 179.9803  1.48302658    04

ORBIT COMPUTED VIA DC_so2rv:
0 REFINED_TLE
1 23802U 0000000  25181.15908569 0.00000087  00000-0  50000-4 0    02
2 23802  79.4556 230.9113 6082740 218.9597 127.6581  1.33460328    00

ORBIT COMPUTED VIA so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA DC_so3rv:
so3rv failed to compute an orbit.

ORBIT COMPUTED VIA RSDC:
0 RSDC_TLE
1 23802U 0000000  25181.13755793 0.00000087  00000-0  50000-4 0    09
2 23802  79.4460 230.9084 6144384 220.8536 111.4785  1.30258492    00
```